

Letter

π^0 -photoproduction on the deuteron via Δ -excitation using the Lorentz Integral Transform

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Abstract. The Lorentz Integral Transform method (LIT) is extended to pion photoproduction in the Δ -resonance region. The main focus lies on the solution of the conceptual difficulties which arise if energy-dependent operators for nucleon resonance excitations are considered. In order to demonstrate the applicability of our approach, we calculate the inclusive cross-section for π^0 -photoproduction off the deuteron within a simple pure resonance model.

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1 Introduction

The Lorentz Integral Transform method (LIT) [1] has been proven to be a powerful technique for calculating inclusive (see, *e.g.*, [2–4] and references therein) as well as exclusive [5–7] photoreaction cross-sections with complete inclusion of final-state interaction (FSI) without calculating final continuum wave functions. Recently, the technique has also been extended to electroweak processes [8].

This success has motivated the extension of the LIT method to pion production processes on light nuclei $A \geq 3$, where existing approaches [9,10] call for considerable improvements. In the case of ^3He , for example, a conventional treatment of FSI would imply a four-body Faddeev-Yakubovsky treatment [11] of the final state which is very complicated. With respect to the present experimental programs to study electromagnetic meson production on light nuclei, *e.g.* at MAMI in Mainz, more sophisticated calculations of such reactions are certainly needed in the near future.

Recently, the LIT has been applied to inclusive pion photoproduction on the deuteron as the simplest possible nuclear target [12]. As a first step, only the near-threshold region has been considered, where only the dominant Kroll-Ruderman term [13] as production operator

was included, and results comparable to traditional approaches were achieved.

In the present work we want to extend this approach to higher photon energies into the $\Delta(1232)$ -resonance region. As we will see below, one cannot proceed naively in a straightforward manner, because of conceptual problems which arise from the energy dependence associated with the resonance contribution to the elementary production operator. We will first give in sect. 2 a brief outline of the LIT approach for energy-independent transition operators. The problem of the standard LIT method for electromagnetic particle production via resonances is discussed in sect. 3, where we also present a formal solution. As a test case, we consider in sect. 4 a simple model for π^0 -production on the deuteron in the Δ -region. The corresponding results, together with a summary and an outlook, are presented in sect. 5.

2 The Lorentz Integral Transform method for energy-independent transition operators

In this section we review briefly the LIT method for inclusive reactions. The central quantity is the response function

$$R(\omega) = \int d\Psi_f |\langle \Psi_f | O | \Psi \rangle|^2 \delta(E_f - E_0 - \omega), \quad (1)$$

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where O is an operator describing the transition from the ground state Ψ , with energy E_0 , to final states Ψ_f , with energies E_f , in the specific process under consideration. In the LIT approach, the response function is not calculated directly. Rather, one first introduces an integral transform of the response function $R(\omega)$ by

$$L(\sigma) = \int_{\omega_{\text{th}}}^{\infty} d\omega \frac{R(\omega)}{(E_0 + \omega - \sigma_{\text{R}})^2 + \sigma_{\text{I}}^2}, \quad (2)$$

where $\sigma = \sigma_{\text{R}} + i\sigma_{\text{I}}$ with $\sigma_{\text{I}} \neq 0$. Furthermore, ω_{th} denotes the reaction threshold.

Inserting the response function from eq. (1), one can rewrite eq. (2) as follows:

$$\begin{aligned} L(\sigma) &= \int dW d\Psi_f \delta(E_f - W) \\ &\times \left\langle \Psi \left| O^\dagger \frac{1}{W - \sigma^*} \right| \Psi_f \right\rangle \left\langle \Psi_f \left| \frac{1}{W - \sigma} O \right| \Psi \right\rangle, \\ &= \int d\Psi_f \left\langle \Psi \left| O^\dagger \frac{1}{E_f - \sigma^*} \right| \Psi_f \right\rangle \left\langle \Psi_f \left| \frac{1}{E_f - \sigma} O \right| \Psi \right\rangle, \end{aligned} \quad (3)$$

with $W = E_0 + \omega$. Now the Schrödinger equation $H|\Psi_f\rangle = E_f|\Psi_f\rangle$ can be used to replace E_f with the Hamiltonian H of the given system

$$L(\sigma) = \int d\Psi_f \left\langle \Psi \left| O^\dagger \frac{1}{H - \sigma^*} \right| \Psi_f \right\rangle \left\langle \Psi_f \left| \frac{1}{H - \sigma} O \right| \Psi \right\rangle. \quad (4)$$

In this step it is essential that the operator O is energy independent. By using the completeness relation

$$\int d\Psi_f |\Psi_f\rangle \langle \Psi_f| = \mathbb{1}, \quad (5)$$

one obtains finally

$$\begin{aligned} L(\sigma) &= \langle \Psi | O^\dagger (H - \sigma^*)^{-1} (H - \sigma)^{-1} O | \Psi \rangle \\ &= \langle \tilde{\Psi} | \tilde{\Psi} \rangle, \end{aligned} \quad (6)$$

where one has introduced the so-called Lorentz state

$$|\tilde{\Psi}\rangle = (H - \sigma)^{-1} O |\Psi\rangle, \quad (7)$$

which obeys an inhomogeneous differential equation

$$(H - \sigma) |\tilde{\Psi}\rangle = O |\Psi\rangle, \quad (8)$$

and which is bound at infinity. Recalling the fact that H is Hermitian and therefore has only real eigenvalues, this feature guarantees a unique solution of (8) because the corresponding homogeneous equation has only the trivial solution. Since the source on the right-hand side of (8) is localized and $\text{Im}\{\sigma\} \neq 0$, the asymptotic behaviour of $\tilde{\Psi}$ at infinity is bound-state-like. Thus, the evaluation of $L(\sigma)$ avoids the explicit calculation of the continuum states Ψ_f but still includes the complete final-state interaction. In the final step, the desired response function R is obtained from $L(\sigma)$ by an appropriate inversion method, see [14] for further details.

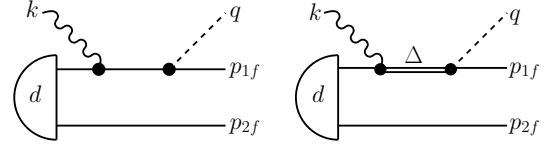


Fig. 1. Nucleon- and Δ -pole contributions to pion photoproduction on the deuteron.

3 The Lorentz Integral Transform method for meson production processes

In the foregoing derivation an essential assumption was that the transition operator O is energy independent. This is, for example, the case for dipole absorption in the long-wavelength limit, but it is certainly no longer fulfilled for a retarded dipole operator. The same is true for the Kroll-Ruderman term in [12]. However, in both cases the energy dependence is smooth and weak. In principle, one could be tempted to replace then in the operator the energy by the corresponding Hamiltonian. But then the equation for the Lorentz state becomes quite complicated and non-linear in the Hamiltonian. In order to avoid this complication, another method has been devised [12] by treating the energy in the operator as a parameter, fixed to some value ϵ . One then determines a Lorentz transform $\tilde{L}(\sigma, \epsilon)$ as a function of this additional variable ϵ . The inversion then yields a response function $\tilde{R}(\omega, \epsilon)$ from which the desired response function is obtained by setting

$$R(\omega) = \tilde{R}(\omega, \omega). \quad (9)$$

The central question is whether this method works also if the production operator contains resonance-like energy dependences which are generated, for example, by the Δ -contribution (see the right diagram of fig. 1). Characteristically, such energy dependence appears in *effective operators*, which take into account the dynamics of projected-out subspaces of the original Hilbert or Fock space. As a pedagogical example and a brief review of the concept of effective operators, we consider a given Hilbert space \mathcal{H} for which one aims at a description within a certain subspace $\mathcal{H}_{\mathcal{P}} = \mathcal{P}\mathcal{H}$, characterized by a projection operator P . The orthogonal complement $\mathcal{H}_{\mathcal{Q}}$ is described by a projection operator Q such that $P^2 = P$, $Q^2 = Q$, $PQ = QP = 0$, and $P + Q = \mathbb{1}$ are fulfilled. Introducing as shorthand $\mathcal{O}_{XY} = X\mathcal{O}Y$ for $X, Y \in \{P, Q\}$ for an arbitrary operator \mathcal{O} , the stationary Schrödinger equation

$$H|\psi\rangle = E_f|\psi\rangle \quad (10)$$

can be cast into a system of coupled equations for the components $|\psi\rangle_{\mathcal{P}} = P|\psi\rangle$ and $|\psi\rangle_{\mathcal{Q}} = Q|\psi\rangle$ which can be written in matrix form according to

$$\begin{pmatrix} H_{\mathcal{P}\mathcal{P}} & H_{\mathcal{P}\mathcal{Q}} \\ H_{\mathcal{Q}\mathcal{P}} & H_{\mathcal{Q}\mathcal{Q}} \end{pmatrix} \begin{pmatrix} |\psi\rangle_{\mathcal{P}} \\ |\psi\rangle_{\mathcal{Q}} \end{pmatrix} = E_f \begin{pmatrix} |\psi\rangle_{\mathcal{P}} \\ |\psi\rangle_{\mathcal{Q}} \end{pmatrix}. \quad (11)$$

We do not need to specify the Hamiltonian H , we only assume that it has the form $H = T + V$, where T is diagonal with respect to $\mathcal{H}_{\mathcal{P}}$ and $\mathcal{H}_{\mathcal{Q}}$, whereas V connects

these two subspaces, *i.e.* $H_{QP} = V_{QP}$. It is then straightforward to derive the following expressions for the components $|\psi\rangle_P$ and $|\psi\rangle_Q$:

$$(T_{PP} + V_{PP}^{\text{eff}}(E_f))|\psi\rangle_P = E_f|\psi\rangle_P, \quad (12)$$

$$|\psi\rangle_Q = (E_f - H_{QQ})^{-1}V_{QP}|\psi\rangle_P \quad (13)$$

with an effective interaction operator

$$V_{PP}^{\text{eff}}(E_f) = V_{PQ}(E_f - H_{QQ})^{-1}V_{QP} + V_{PP}. \quad (14)$$

This interaction is just one example of a so-called *effective* operator $\mathcal{O}^{\text{eff}}(E_f)$, which by construction is acting solely in \mathcal{H}_P .

The concept of effective operators leads therefore to a considerable simplification for the solution of the Schrödinger equation in the subspace \mathcal{H}_P . The price one has to pay is that the corresponding Hamiltonian H_{PP}^{eff} (14) becomes energy dependent and therefore *non-Hermitian*. Moreover, one should keep in mind that the effective operator is well defined only if the energy spectrum of H_{QQ} has no overlap with E_f of the effective Hamiltonian in \mathcal{H}_P . Otherwise it will become singular. This remark indicates the limitation of this concept, which poses a serious problem with respect to the LIT, because the latter needs the information on the whole spectrum of the Hamiltonian which overlaps with the spectrum of H_{QQ} . This will be illustrated below.

A prominent example is given by the one-pion exchange potential (OPEP) of NN -scattering. In this case one deals with a Fock space \mathcal{F} since the number of pions is not fixed, *i.e.* one deals with configurations consisting of two nucleons and zero, one, two etc. pions. In the simplest case, one restricts the possible configurations to NN - and πNN -components, denoting the corresponding subspaces by \mathcal{F}_P and \mathcal{F}_Q , respectively. The latter is needed for generating the effective NN -force via one-pion exchange. The Hamiltonian consists of a diagonal kinetic energy T and a πN -vertex for V_{PQ} , generating transitions $\mathcal{F}_P \leftrightarrow \mathcal{F}_Q$. In this case V_{PP} is set to zero. Therefore, V_{PP}^{eff} in (14) is just the well-known retarded one-pion exchange potential (OPEP). Due to its energy dependence, the effective Hamiltonian $T_{PP} + V_{PP}^{\text{eff}}$ is not Hermitian. Moreover, above the pion production threshold it becomes singular due to the competing states in \mathcal{F}_Q , namely in πNN -space. This energy dependence usually is neglected, if one is solely interested in NN -scattering below the pion threshold. A further consequence is the appearance of an electromagnetic two-body pion exchange current.

We now turn to pion photoproduction and consider the nucleon-pole diagram of fig. 1. Here the intermediate NN -configuration is projected out in favor of an effective operator. Thus, in this case the meaning of the projectors P and Q have to be interchanged, *i.e.* \mathcal{F}_P is identified with the πNN -space and \mathcal{F}_Q with the NN -space. The effective photoproduction current operator

$$O(E_f) \propto \frac{1}{E_f + i\varepsilon - T_{QQ}}, \quad (15)$$

contains a pole structure at all possible energies in contrast to the above effective OPEP, which cannot be transformed away. A similar problem occurs if resonance degrees of freedom are included, see as an example the Δ -pole diagram in fig. 1. Therefore, any realistic effective pion production operator (see, for example, [15, 16]) is energy dependent due to the occurrence of intermediate NN - or $N\Delta$ -components. As already alluded to above, this energy dependence leads to severe problems in the LIT approach. Recalling the different steps in eqs. (4) and (6), and applying the LIT approach naively would mean to replace the energy E_f in (15) by the full Hamiltonian H of the final πNN -system. This, however, does not make any sense, because H_0 and H act on different particle systems. Also the method of treating the energy as a parameter does not work, because in this case the effective operator becomes singular and thus the Lorentz state is not normalizable any more. The only possible way out of this dilemma is to avoid the use of effective operators by going back to the original Fock space \mathcal{F} containing configurations with different number of particles. In \mathcal{F} the basic interactions, *e.g.* πN -vertices and currents, are energy independent and Hermitian so that the above-mentioned problems will not occur.

In order to illustrate the procedure, we will first consider a general Fock space \mathcal{F} consisting of N orthogonal subspaces labeled by l . The corresponding projectors P_l fulfill the relations

$$\mathbb{1} = \sum_{l=1}^N P_l \quad \text{with} \quad P_l P_m = \delta_{lm} P_l. \quad (16)$$

The full Fock space Hamiltonian $H = H_0 + V$ is assumed to consist in a diagonal part H_0 and an interaction $V_{lm} = P_l V P_m$, with $l, m \in \{1, \dots, N\}$, allowing transitions between the various subspaces. A transition operator, describing an electromagnetic process, is generally expressed by a Hermitian current operator j^μ in \mathcal{F} , which does not contain any resonance-like energy dependence,

$$O = \sum_{l,m=1}^N \varepsilon_\mu j_{lm}^\mu \quad \text{with} \quad j_{lm}^\mu = P_l j^\mu P_m, \quad (17)$$

where ε_μ denotes the photon polarization vector. A remaining trivial smooth energy dependence via current structures depending on the photon momentum can be handled by the parameter method.

In this Fock space, the LIT equation reads now

$$(H - \sigma)|\tilde{\Psi}\rangle = O|\Psi\rangle, \quad \text{Im}\{\sigma\} \neq 0. \quad (18)$$

To solve this equation we use the method of resolvents in analogy to standard scattering theory, *i.e.* we introduce

$$\mathcal{G}_0(\sigma) = \frac{1}{\sigma - H_0}, \quad \mathcal{G}(\sigma) = \frac{1}{\sigma - H}. \quad (19)$$

Defining a conventional T -matrix by $T\mathcal{G}_0 \equiv V\mathcal{G}$, one then obtains the following standard equations for the \mathcal{G} - and

T -matrices

$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0 T \mathcal{G}_0 = \mathcal{G}_0 + \mathcal{G}_0 V \mathcal{G}, \quad (20)$$

$$T = V + V \mathcal{G}_0 T. \quad (21)$$

A formal solution of the LIT equation can be written as

$$|\tilde{\Psi}\rangle = -\mathcal{G}O|\Psi\rangle. \quad (22)$$

Since O is regular and $\text{Im}\{\sigma\} \neq 0$, the Lorentz state $\tilde{\Psi}$ is normalizable like a bound state.

4 Application to π^0 -photoproduction via the excitation of the Δ -resonance

As a test case, we will consider now π^0 -photoproduction off the deuteron in the Δ -region. Then \mathcal{F} comprises besides $\pi^0 NN$ - also NN - and $N\Delta$ -states, so that nucleon-pole as well as Δ -pole contributions to the photoproduction process can be taken into account.

Extending the compact matrix notation (11) with respect to the three projectors P_π , P_N and P_Δ for the $\pi^0 NN$ -, NN - and $N\Delta$ -subspaces, respectively, we can cast kinetic and interaction parts of the Hamiltonian $H = T + V$ into the following forms:

$$H_0 = \begin{pmatrix} T_N & & \\ & T_\Delta & \\ & & T_\pi \end{pmatrix}, \quad V = \begin{pmatrix} V_{NN} & V_{N\Delta} & V_{N\pi} \\ V_{\Delta N} & V_{\Delta\Delta} & V_{\Delta\pi} \\ V_{\pi N} & V_{\pi\Delta} & V_{\pi\pi} \end{pmatrix}. \quad (23)$$

Transitions between different subspaces are described by the non-diagonal matrix elements of V . For example, $V_{\pi N}$ contains the $\pi^0 N$ -vertex allowing transitions between the NN - and $\pi^0 NN$ -states. Since we only want to demonstrate the applicability of our approach and do not aim at a quantitative description of the data, we restrict the interaction solely to $V_{\Delta\pi}$ and V_{NN} , *i.e.* we use

$$V = \begin{pmatrix} V_{NN} & 0 & 0 \\ 0 & 0 & V_{\Delta\pi} \\ 0 & V_{\pi\Delta} & 0 \end{pmatrix}. \quad (24)$$

The parametrization of the $\Delta N \pi^0$ -vertex $V_{\Delta\pi}$ is taken from [17]. As electromagnetic current, we consider here solely the dominant $M1$ - $N\Delta$ -current $\mathbf{j}_{N\Delta}$:

$$\mathbf{j}_{\Delta N}(\mathbf{k}) \sim \frac{G_{M1}^{0\Delta N}}{2M_N} i \boldsymbol{\sigma}_{\Delta N} \times \mathbf{k} \quad (25)$$

with $G_{M1}^{0\Delta N} = 4.22$. With these building blocks we can describe the dominant Δ -pole contribution to pion photoproduction. In view of the approximations (24) and (25), the NN -interaction V_{NN} contributes solely to the deuteron ground state. For reasons of simplicity, we have used a pure S -wave Yamaguchi potential [18] with modern parameters from [19]. For a shorter notation we label operators A connecting the same subspace with only one index,

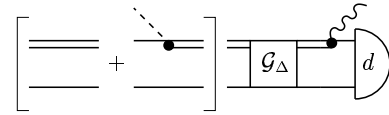


Fig. 2. Diagrammatic representation of $|\tilde{\Psi}\rangle$ in eq. (26).

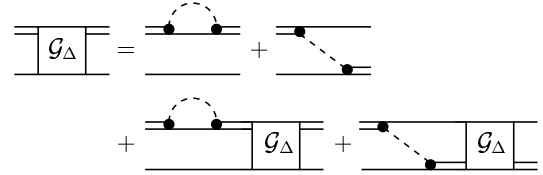


Fig. 3. Diagrammatic representation of \mathcal{G}_Δ in eq. (27).

i.e. $A_{ll} \equiv A_l$. The Lorentz state can then be written as (see fig. 2)

$$-|\tilde{\Psi}\rangle = \mathcal{G}O_{\Delta N}|\Psi\rangle = (\mathbb{1}_\Delta + \mathcal{G}_{0\pi}V_{\pi\Delta})\mathcal{G}_\Delta O_{\Delta N}|\Psi\rangle. \quad (26)$$

The resolvent \mathcal{G}_Δ fulfills the equation (see fig. 3)

$$\mathcal{G}_\Delta = \mathcal{G}_{0\Delta} + \mathcal{G}_{0\Delta}V_{\Delta\pi}\mathcal{G}_{0\pi}V_{\pi\Delta}\mathcal{G}_\Delta, \quad (27)$$

which allows one to rewrite the LIT into the following form:

$$L = -\frac{1}{\sigma_I} \text{Im} \{ \langle \Psi | O_{N\Delta} \mathcal{G}_\Delta O_{\Delta N} | \Psi \rangle \}. \quad (28)$$

V_Δ consists of a loop diagram (disconnected) and a genuine two-body one-pion exchange potential (connected), see fig. 3. If the latter is neglected, we will refer to it as impulse approximation (IA) and denote the corresponding resolvent from here on as $\mathcal{G}_\Delta^{\text{IA}}$. Therefore, in impulse approximation L^{IA} is given by (28) where \mathcal{G}_Δ is replaced by

$$\mathcal{G}_\Delta^{\text{IA}} = \frac{1}{\sigma - T_\Delta - \Sigma_\Delta(\sigma)}, \quad (29)$$

with Σ_Δ as self-energy of the Δ

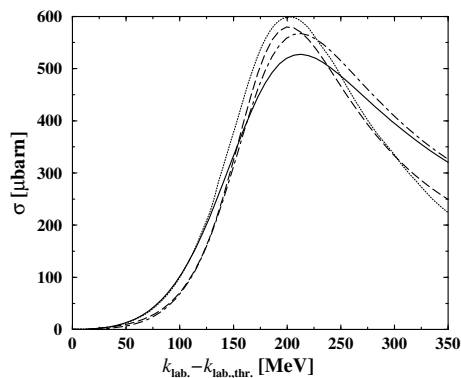
$$\Sigma_\Delta(\sigma) = V_{\Delta\pi}\mathcal{G}_{0\pi}(\sigma)V_{\pi\Delta}|_{\text{disconnected}}. \quad (30)$$

5 Results and conclusion

The foregoing formalism has been applied to π^0 -photoproduction on the deuteron. We have calculated eq. (28) using a partial-wave decomposition in the $N\Delta$ -space. We have included all partial waves up to a total angular momentum $J_{\text{max}} = 5$ which are listed in table 1. Although we take all listed partial waves into account, convergence is in practice already fulfilled for $J_{\text{max}} = 2$, because the dominant contributions come from the S -waves (${}^3S_1(N\Delta)$, ${}^5S_2(N\Delta)$). The resulting response function, shown in fig. 4, turns out to be very stable numerically, which was tested by using different inversion techniques as discussed in [14] as well as different σ_I in the range between 5 and 50 MeV producing almost indistinguishable results. The result shown in fig. 4 has been calculated

Table 1. $N\Delta$ -partial waves with total angular momentum J and parity π up to $J_{\max} = 5$.

J	π	$^{2S+1}L_J$
0	+	5D_0
0	-	3P_0
1	+	$^3S_1, ^3D_1, ^5D_1$
1	-	$^3P_1, ^5P_1, ^5F_1$
2	+	$^5S_2, ^3D_2, ^5D_2, ^5G_2$
2	-	$^3P_2, ^5P_2, ^3F_2, ^5F_2$
3	+	$^3D_3, ^5D_3, ^3G_3, ^5G_3$
3	-	$^5P_3, ^3F_3, ^5F_3, ^5H_3$
4	+	$^5D_4, ^3G_4, ^5G_4, ^5I_4$
4	-	$^3F_4, ^5F_4, ^3H_4, ^5H_4$
5	+	$^3G_5, ^5G_5, ^3I_5, ^5I_5$
5	-	$^5F_5, ^3H_5, ^5H_5, ^5J_5$

**Fig. 4.** Total cross-section of π^0 -photoproduction on the deuteron. Notation of the curves: dashed: IA with constant Δ -mass; dash-dotted: IA using eq. (29); full: inclusion of $N\Delta$ -interaction; dotted: IA according to [20] (only resonance).

using $\sigma_I = 20$ MeV and the inversion has been obtained using the so-called Fridman method [14]. For comparison, the IA is evaluated according to (29) with a variable σ -dependent Δ -mass without using a partial-wave decomposition. In addition, we have calculated another IA with a static Δ -mass ($M_\Delta = 1232$ MeV) for a better comparison with the IA of [20].

One readily notes that the $N\Delta$ -interaction has a sizeable influence near threshold leading to an enhancement and is still moderate near the maximum resulting in a lowering of the absolute size by about 8%. Certainly, for a realistic description of the cross-section in the Δ -region one has to include besides non-resonant photoproduction contributions the neglected FSI, *i.e.* NN - and πN -interactions [21, 22]. However, we have chosen the deuteron only for the purpose of illustrating in a simple

manner the problems one encounters in applying the LIT method for such a process and the way out. If one goes to higher energies above the Δ -region, further resonances have to be included with a corresponding enlargement of the Fock space.

In conclusion, we have demonstrated how to extend the LIT method when energy-dependent effective operators are involved. It turns out that in such a case those states, which have been projected out in favor of effective operators, have to be included explicitly in an expanded Fock space. The latter should contain all relevant degrees of freedom in the considered energy regime, *i.e.* nucleons, pions and the Δ -resonance for the specific process of pion photoproduction on the deuteron up to about 500 MeV excitation energy. Within a very simplified approach, neglecting for example non-resonant Born contributions, we have shown that this concept is successful. In future work, it should be extended with respect to the following aspects: First of all, additional FSI as well as non-resonant Born contributions have to be included so that a comparison with already existing experimental data [23] makes sense. Moreover, in analogy to [7], an extension to exclusive reactions, *i.e.* coherent as well as incoherent pion production, should be performed. Furthermore, this method should really pay off for reactions on more complex nuclei, namely meson photoproduction on other light nuclei like ^3He or ^4He .

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